
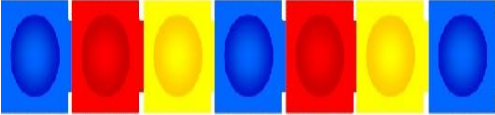


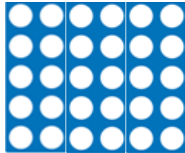
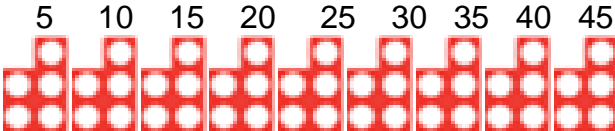

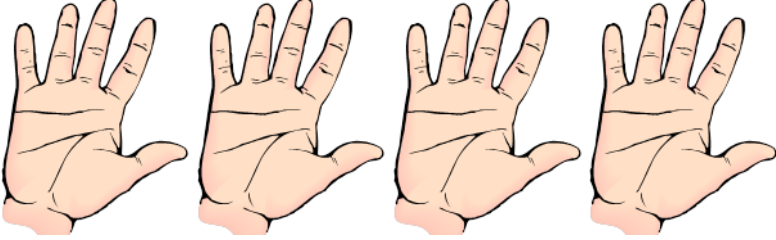







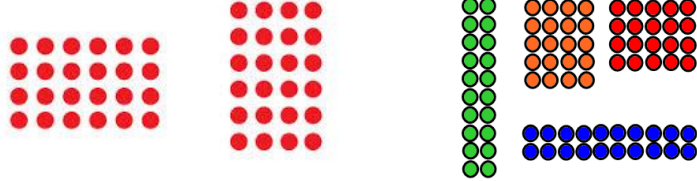
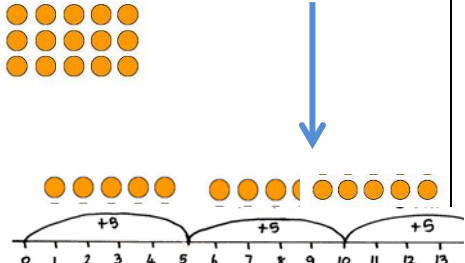
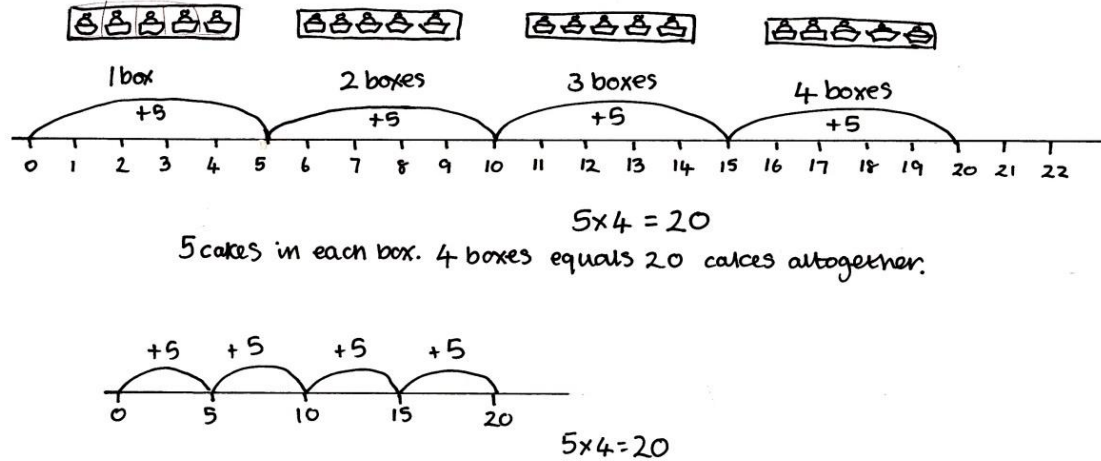
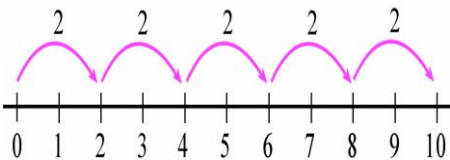
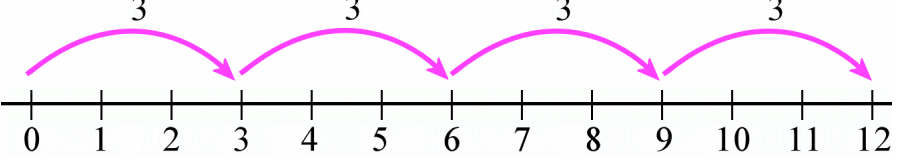
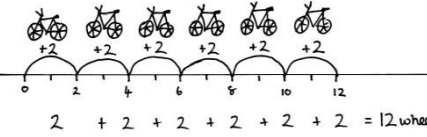
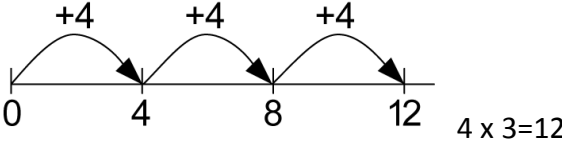


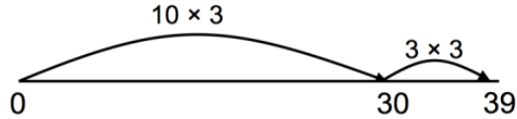


Step	Skill	Number Range	Possible Models / Images	Examples
1.	Talk about, recognise continue and recreate simple patterns using practical apparatus.	Within 10 - 20	Beads/ multilink columns./numicon/ pattern in the environment-counting in 2s, 3s or 5s depending on pattern observed.	  <p>Spot repeating patterns in environment. Recreate similar patterns from concrete materials. Count in 2s to find out how many.</p> 
2.	<p>Count repeated groups of the same size 2s, 5s, 10s. Introduce multiplication sign. First number is the multiplier and the 2nd is the multiplicand (the one that is doing the action to the other).</p>	<p>Real life imagery, moving on to mathematical images eg numicon How many fingers? 5 multiplied 4 times (5 x 4)</p>		<p>Written numbers and symbols are visible and scribed alongside so that the child is learning from the concrete.</p> <p>$5 \times 9 = 45$ (5 multiplied 9 times is 45)</p>  <p>$10 \times 3 = 30$</p> <p>(10 multiplied 3 times=30)</p> 

Step	Skill	Number Range	Possible Models / Images	Examples
3.	Continue to count in multiples. Show 5 fingers on each hand to count in 5's and 10', continue the pattern, realising the repeat of 5/0.	Within 100.		 <p style="text-align: center;">5 10 15 20 25 30</p>
4.	Iconic jottings (dots in sets) to find out how many.			 <p style="text-align: center;">3 + 3 + 3 + 3</p> <p style="text-align: center;">3 multiplied 4 times (3 x 4)</p> <p>At this point, children should also compare this to 4 x 3 to appreciate x is commutative.</p>  <p style="text-align: center;">4 multiplied 3 times = 12 (4 x 3 = 12)</p>

Step	Skill	Examples	Possible Models / Images												
5.	<p>Children begin to see the merit in organising their jots in squares / rectangles to facilitate counting (link to numicon and encourage children to record their 'dots' in a numicon layout). This helps visualise the patterns of numbers.</p>		 <p style="text-align: right;">4 x 5</p> <p>Their jottings link to model images of arrays. The 2 images go 'hand in hand'.</p> <div style="text-align: center;">  </div>												
6.	<p>Use arrays for multiplication.</p> <p>Use number squares to visualise the patterns of numbers.</p> <p>Know that multiplication is commutative.</p>	<p>$3 \times 4 = 12$</p>  <div style="text-align: center;">  </div> <table border="1" data-bbox="524 979 853 1219"> <tr> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>7</td> <td>8</td> <td>9</td> </tr> <tr> <td>10</td> <td>11</td> <td>12</td> </tr> </table>	1	2	3	4	5	6	7	8	9	10	11	12	 <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Here are 20 counters. How could you arrange them in equal rows? How could you use a number sentence to show your arrangement?</p> </div> <p>Arrays are a visual representation of multiplication where children are taught to identify groups or sets of numbers e.g. 'four lots of 6' and 'six lots of 4'. Practical resources should be used to create images.</p> <p>Children should know that multiplication is commutative and 4×6 has the same answer as 6×4.</p> <p>Using arrays will support the development of the grid method and make links to division.</p>
1	2	3													
4	5	6													
7	8	9													
10	11	12													

Step	Skill	Examples	Possible Models / Images
7.	<p>Multiplying through making links to counting on in steps of equal size. (multiples of 2's, 5's and 10's, then other numbers).</p>	<p>Make links to the physical objects with the repeated counting on a structured number line. E.g. convert the array to a number line by cutting it up into strips and placing above the number line (5x3).</p> 	
8.	<p>Use a structured number line for repeated addition, counting in equal steps.</p> <p>Know that multiplication is commutative.</p>	<p>2 multiplied 5 times (2 x 5) is the same as</p> 	 <p>$3 \times 4 = 12$ and can be shown as $3 + 3 + 3 + 3$.</p> <p>Children need to see that this gives the same answer as $4 \times 3 : 4 + 4 + 4 = 12$.</p>

Step	Skill	Number Range	Possible Models / Images	Recording/ Examples
9.	Once secure in multiplication using arrays and a structured number line, move on to counting in equal steps on an unstructured number line.		<p>If I have 6 bicycles, how many wheels would there be?</p> 	 <p>4 x 3 = 12</p> <p>Show commutative aspect:</p>  <p>3 x 4 = 12</p>
10.	Moving towards use of understanding of place value and partitioning. Children partition numbers into tens and ones and multiply by a single digit.		<p>When the numbers become too big, arrays and number lines become unwieldy and children use place value knowledge to multiply the tens then the ones.</p> <p>Children must be confident in multiplying by 10 and understand that when numbers are multiplied by 10 they move a decimal place to the left.</p>	<p>16 x 3 = 78 (10 x 6) + (6 x 3) 60 + 18</p> <p>An array or number line may be used to support part of this process where children are not confident in multiplication tables e.g.</p> <p>16 x 4 = 84 (10 x 4) + (6 x 4) 40 + 24</p>  <p>Children may need to draw an array to work out the multiplication for the ones digit x ones digit as shown here for 6 x 4.</p> <p>Children should also be encouraged to try other strategies where appropriate e.g. in this case doubling and doubling again.</p>
11.	Use a number line to count in groups of multiples.	Teens number by a single digit e.g. 14 x 7		<p>13 threes can be made from 10 threes and 3 threes.</p>  <p>Children may prefer to link this to an unstructured number line. Children use their knowledge of multiplication facts to count in groups of multiples.</p>

12.	Use the grid method for short multiplication.	2 digit numbers x 1 digit number TO x O	→	<p>Once children are confident with times table knowledge, related facts and partitioning, they should be taught to use the grid method of multiplication for multiply a 2 digit number by a single digit (starting with teen numbers).</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $\begin{array}{r l} \text{T} & \text{O} \\ 3 & 5 \times 7 \\ \hline 30 & 5 \end{array}$ </div> <div style="text-align: center;"> <table border="1" style="border-collapse: collapse;"> <tr> <td style="padding: 5px;">X</td> <td style="padding: 5px;">30</td> <td style="padding: 5px;">5</td> </tr> <tr> <td style="padding: 5px;">7</td> <td style="padding: 5px;">210</td> <td style="padding: 5px;">35</td> </tr> </table> <p>210 + 35 = 245</p> </div> </div> <p>The larger number should be partitioned across the top of the grid.</p>	X	30	5	7	210	35
X	30	5								
7	210	35								
13.	Use the expanded column method for short multiplication.	2/3 digit numbers x 1 digit number. TO x O HTO x O	→	<p>The next step is to represent the method of recording in a column format, but showing the working. Children should be able to see the links with the grid method above. Children need to be taught to set their calculations out in columns, paying particular attention to the place value of each number. It may be helpful for the children to label each place value column above the calculation.</p> <div style="display: flex; justify-content: space-between; align-items: flex-start;"> <div style="text-align: center;"> $\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 38 \\ \times 7 \\ \hline 56 \quad (7 \times 8) \\ 210 \quad (7 \times 30) \\ \hline 266 \end{array}$ </div> <div style="border: 1px solid black; padding: 10px; width: 200px;"> <p>This method requires children to have instant recall of multiplication facts, understanding of multiples of ten, and understanding of the column method for addition.</p> </div> </div> <p>Children should describe what they do by referring to the actual values of the digits in the columns. For example, the second step in 38×7 is 'seven multiplied by thirty', not 'seven times three'.</p>						

Step	Skill	Number Range	Possible Models / Images	Recording/ Examples									
14.	Use the compact column method for short multiplication .	2 and 3 digit numbers x 1 digit number. TO x O HTO x O	→	<p>Once confident with the expanded method, children are now ready to be taught the compact method for multiplication. Digits that are carried should be recorded below the line, in the correct place value column. Multiply the ones first, moving towards the highest integer.</p> 123×5 <div style="display: flex; justify-content: space-around; text-align: center;"> <div> <p>1st Step</p> $\begin{array}{r} 123 \\ \times 5 \\ \hline 615 \end{array}$ </div> <div> <p>2nd Step</p> $\begin{array}{r} 123 \\ \times 5 \\ \hline 15 \\ 11 \end{array}$ </div> <div> <p>3rd Step</p> $\begin{array}{r} 123 \\ \times 5 \\ \hline 615 \end{array}$ </div> </div>									
15.	Use the grid method for long multiplication .	2 digit numbers x 2 digit numbers. TO x TO	→	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>20</td> <td>600</td> <td>100</td> </tr> <tr> <td>6</td> <td>180</td> <td>30</td> </tr> </table> $600 + 100 = 700$ $180 + 30 = 210$ $700 + 210 = 910$ <p>Recombining the answers can be represented as column addition if this matches the addition progression stage.</p>	x	30	5	20	600	100	6	180	30
x	30	5											
20	600	100											
6	180	30											

Step	Skill	Number Range	Possible Models / Images	Recording/ Examples
16.	Use the compact column method for long multiplication .	2 and 3 digit numbers x 2 and 3 digit numbers. TO x TO HTO x TO HTO x HTO	→	<div style="text-align: center;"> 123×45 </div> <p>1st step</p> $\begin{array}{r} 123 \\ \times 45 \\ \hline 615 \\ \hline \end{array} \quad (5 \times 123)$ <p>2nd step</p> $\begin{array}{r} 123 \\ \times 45 \\ \hline 615 \\ \hline 0 \end{array} \quad \text{Place value holder to show we are multiplying tens.}$ <p>3rd step</p> $\begin{array}{r} 123 \\ \times 45 \\ \hline 615 \\ + 4920 \\ \hline 5535 \end{array}$ <p>(5 x 123) (40 x 123)</p>

The most important concept at this stage is for the children to understand the importance and use of the **place value holder**.